**Excitations in B field w/ smooth E field**

Now we’d like to see how disorder affects the energy levels of a particle in a magnetic field. This is a complicated task in general, and so we’re going to take it slow, examining how adding simple electric fields affects the energies. And then we’ll try to extrapolate to more complicated situations.

**Particle in B field (Landau gauge) and linear electric potential**

So say we have our particle in a vertical magnetic field B, described in the Landau gauge as **A** = Bx, and then also put in an electric field **E** = E. Then we have:



[e carries sign] Then we still have HO oscillator states, ultimately. For instance, let ψ = exp(ikyy)ψ(x). Then,



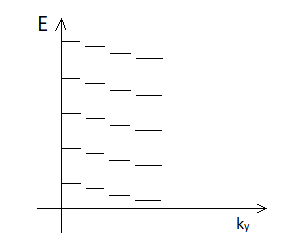
Okay so finally,



This is the effectively the usual HO equation. Eigenfunctions and eigenenergies will be (note our equation implies we have ε + (1/2)m(E/B)2 + (E/B)ky = ωB(n+1/2)…, not the other way around…took me 2 days to realize this 😐, so solving for ε…)



where ωc = ωB = |e|B/m. The degeneracy of the Landau levels is lifted now, as there is term proportional to ky. So energy is decreasing rightward, with increasing ky,



Would like to note that we can write the states and energy as:



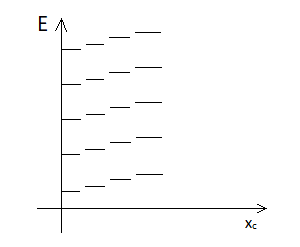
where,



and xc is the location of the center of the wavefunction along the x-axis (the peak for n = 0, and midway between the two peaks for n = 1, etc.), and vdrift is the average (physical) velocity of the wavefunction, and Φ = -eEx of course. Note that the formula, in this form, states that the energy of the particle is sum of oribital kinetic energy around the B field, drift kinetic energy in the direction of E×B, and electric potential energy. This makes sense from a classical physics perspective. And see the EM folder/Charge dynamics file for a derivation of how the motion of a charge initially at rest does indeed develop into the sum of precisely these orbital and drift motions. Gonna prove the energy formula works…



So that checks out. We can interpret the last two terms in the new energy formula as the obvious bulk potential and kinetic energy. The first would be the kinetic energy associated with the particle’s diamagnetic orbital motion about the magnetic field (the particle’s orbital radius is ~ μm, as per our rough calculation in the QM/Time Independent/Symmetric gauge file calculation) and fuzzy radial kinetic energy. And so we can say that energy is increasing rightward with xc (presuming negative e)



How many states are in a Landau level? Should be the same as before. Apropos ky, we must have periodicity of wavefunction in y direction, and this restricts ky to 2πq/Ly  (q being integer quantum number here). And so xc is restricted to the discrete values,



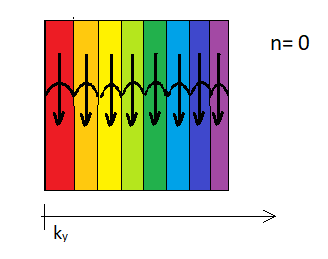
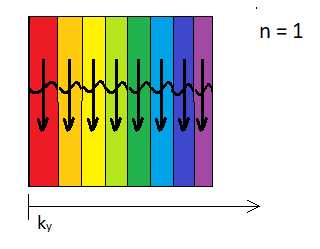
Requiring that xc remain within the sample width of Lx yields the usual Landau level degeneracy formula:



(ky would take on negative values for negative charges and positive values for positive charges – but ky is unphysical so no problem) Note also, that each state now has a non-zero velocity expectation. Moreover it is in the y-direction, which is consonant with the classical picture that the drift velocity of a charge in an **E**, **B**, field is in the direction of **E**×**B**. We can calculate the expectation of this drift velocity. Note **v** = (**p** - e**A**)/m.



So interesting that each state has the same expected velocity. Also interesting that this corresponds to the classical drift velocity in an EB field. So now we can picture the states as doing something like:

Let’s do another example.

**Particle in B field (symmetric gauge) and simple central electric potential**

Going to try a more complicated electric potential. And we want to ultimately show what we did in the previous example, that the energy of a particle in a magnetic field is approximately:



i.e., its energy consists of orbital kinetic energy about the magnetic field lines, drift velocity kinetic energy in the direction of E×B, and electric potential energy within the electric field. To facilitate this we’ll use the symmetric gauge, like was done in the QM folder (Time-Independent). So, to succinctly review, we start with:



And then we use the symmetric gauge.



and we’re going to use a more complicated electric potential than last time:



In terms of this, in cylindrical coordinates, our equation reduces to (see that QM file for details):



We can postulate, as we did, that ψ = ei(m\_ℓ)φR(r), in which case we’ll get:



And putting in WKB form,



where,



This turns out to be the simplest possible non-trivial choice that we can make. Instead of doing this out for real, let’s use the WKB approximation – I’m pretty sure the WKB approximation actually gives us the exact results. So looking back at the 2D WKB approximation in the QM time-independent file, we have our energy levels equation (remember we’re setting ℏ = 1),



where A1 and A2 are the turning points and,



Filling in V(r), we have to evaluate,



And can see why we chose that potential – because it doesn’t change the overall form of the integrand. We’ll want to do like we did in the 2D WKB QM file, and clean this up by defining some propitious units. First we’ll write:



And let’s define a new magnetic length, ℓ´B. The r2 potential term seems a good candidate to define this. So apropos this term, we’ll have once we factor out the energy unit a term underneath r2 which must be of length dimensions:



where,



and then apropos the 1/r2 term, let’s define a new magnetic quantum number, α. So we’ll define α via:



where,



Then our integral becomes,



Now change variables to ξ = r/ℓ´B,



So now we have:



Now let’s define:



and we’ll have,



This form matches that which we had in the QM folder/2D WKB file. So we can proceed with that file to to work out what the LHS integral is. First, define ξ = √z. Then we’ll have:



Now z1,2 are the turning points of our integrand, i.e., where it equals zero. These are at:



So we can say,



From that file, we know the integral equals:



So we have:



So then filling ε´ and α´



And putting units back in,



Now we’d like to see if we can make the same semiclassical interpretation of the energy as we did in the previous example. First we gotta work out where the peaks are. From same QM 2D WKB file, we had:



Would like to primarily see where wavefunction peaks are. This is between the turning points. And these were,



Continuing,



For large-ish α, we have:



The center of the wavefunction would be at:



And in terms of actual position,



And let’s work out where rc is for weak fields:



(we can neglect the C1,2 terms because they will be multiplying C’s already, in our calculation below) Now we saw in the previous example that the energy spectrum had a very classical interpretation, of quantum kinetic energy + classical kinetic and potential energy. Does the same hold here? Doesn’t look like it does exactly. But let’s expand for weak electric field, i.e., small C1,2,3, and also large mℓ. Then,



And proceeding,



keeping with approximation that |mℓ| >> 1, and η, we have:



Then must check,



So this checks out, and we can write, for weak fields:



The first term is exactly the energy levels of a particle in a magnetic field, in the symmetric gauge (see QM folder and recall it’s equivalent to ωB(n+1/2)). And the second term is the potential energy term coming from the electric field, of course. We’re missing the (1/2)mvdrift2 term that we had in the first example, but this would require going out to second order in the C’s, and I don’t want to do all that.